# Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2019

November 8, 2019

# Part I

# A. STATISTICS

• Numbers and percentages in each class.

See Table 1, page 1.

• Numbers of vivas and effects of vivas on classes of result. As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

#### • Marking of scripts.

The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

• Numbers taking each paper.

See Table 7 on page 9.

			Number	ſ		Percentages %				
	2019	(2018)	(2017)	(2016)	(2015)	2019	(2018)	(2017)	(2016)	(2015)
Ι	58	(53)	(48)	(44)	(45)	57.43	(56.99)	(57.14)	(50.57)	(46.39)
II.1	40	(26)	(23)	(31)	(39)	39.6	(27.96)	(27.38)	(35.63)	(40.21)
II.2	2	(13)	(12)	(9)	(13)	1.98	(13.98)	(14.29)	(10.34)	(13.4)
III	1	(1)	(1)	(3)	(0)	0.99	(1.08)	(1.19)	(3.45)	(0)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	101	(93)	(84)	(87)	(97)	100	(100)	(100)	(100)	(100)

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# B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

### C. Notice of examination conventions for candidates

The first notice to candidates was issued on 12th February 2019 and the second notice on 1st May 2019. These contain details of the examinations and assessments.

All notices and the examination conventions for 2019 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

# Part II

# A. General Comments on the Examination

The examiners would like to thank in particular Gemma Proctor, Waldemar Schlackow and Charlotte Turner-Smith for their commitment and dedication in running the examination systems. We would also like to thank Nia Roderick, and the rest of the Academic Administration Team for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Richard Jozsa and Dr Jonathan Woolf for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting.

#### Timetable

The examinations began on Monday 3rd June and finished on Wednesday 19th June.

#### Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the board (the 'Mitigating Circumstances Panel') had a preliminary meeting to discuss the individual notices to examiners at Part C. There were 3 notices, which the panel classified in bands 1, 2, 3 as appropriate. The full board of examiners considered the 3 cases in the final meeting. All candidates with certain conditions (such as dyslexia, dyspraxia, etc.) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks.

#### Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Gemma Proctor, Charlotte Turner-Smith and Hannah Harrison, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

#### **Determination of University Standardised Marks**

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers  $N_1$ ,  $N_2$  and  $N_3$  are first computed for each paper:  $N_1$ ,  $N_2$  and  $N_3$ are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges [70, 100], [60, 69] and [0, 59], respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map  $R \to U$  (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points (100, 100),  $P_1 = (C_1, 72), P_2 = (C_2, 57), P_3 = (C_3, 37), \text{ and } (0, 0)$ . The values of  $C_1$  and  $C_2$  are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by  $N_1$  and  $N_2$ , is the same as the I and II.1 proportion of USMs achieved on the paper. The value of  $C_3$  is set by the requirement that  $P_2P_3$  continued would intersect the U axis at  $U_0 = 10$ . Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points  $P_1, P_2, P_3$  by hand, so as to alter the map raw  $\rightarrow$  USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper,  $P_1$ ,  $P_2$ ,  $P_3$  are the (possibly adjusted) positions of the corners above, which together with the end points (100, 100) and (0,0) determine the piecewise linear map raw  $\rightarrow$  USM. The entries  $N_1$ ,  $N_2$ ,  $N_3$  give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of  $P_1$ ,  $P_2$ ,  $P_3$ .

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C1.1	(11, 37)	(29, 57)	(41, 70)		1	6	2
C1.2	(7, 37)	(26, 57)	(36, 70)		1	5	2
C1.3	(10, 37)	(27.3, 57)	(40.8, 72)		5	10	1
C1.4	(9.65, 37)	(20, 57)	(37.8, 72)		7	7	2
C2.1	(11.71, 37)	(20.4, 57)	(30, 72)		11	1	0
C2.2	(10, 37)	(25, 57)	(34.8, 72)		15	3	1
C2.3	(7.35, 37)	(21, 57)	(33.8, 72)		3	1	0
C2.4	(13, 37)	(26.3, 57)	(39.8, 72)		7	1	2
C2.5	(14.59, 37)	(23, 57)	(31, 72)		6	1	1
C2.6	(8.44, 37)	(14.7, 72)	(21, 70)		9	0	0
C2.7	(14, 37)	(26.8, 57)	(32.8, 70)		17	4	2
C3.1	(5, 37)	(17, 57)	(28, 72)		11	1	0
C3.2	(12.12, 37)	(21.1, 57)	(30, 72)		5	3	2
C3.3	(14.01, 37)	(20, 57)	(37, 72)		4	4	1
C3.4	(8.15, 37)	(19, 57)	(38.2,72)		14	1	0
C3.5	(11, 37)	(25.2, 57)	(34.2, 72)		6	3	1
C3.7	(8.1, 37)	(18, 57)	(33.6, 72)		10	5	0
C3.8	(8, 37)	(17, 57)	(37, 72)		11	6	0
C3.10	(8, 37)	(14, 57)	(27, 72)		9	6	0
C4.1	(9, 37)	(15, 57)	(28, 72)		9	3	0
C4.3	(14.82, 37)	(23, 57)	(31.8, 72)		3	1	0
C4.4	(5, 37)	(15, 57)	(23, 72)		0	2	0
C4.6	(13.04, 37)	(22.7, 57)	(37, 72)		2	0	0
C4.8	(12.35, 37)	(21.5, 57)	(30.5, 72)		1	2	0
C5.1	(5.1, 37)	(18, 57)	(32, 72)		5	15	2
C5.2	(6.08, 37)	(23, 57)	(32, 72)		5	11	1
C5.5	(8.9, 37)	(15.5, 57)	(38, 72)		9	21	1
C5.6	(9, 37)	(18.3, 57)	(41, 72)		8	14	1
C5.7	(7.52, 37)	(22, 57)	(42, 72)		6	11	0
C5.9	(9.88, 37)	(22.5, 57)	(34, 72)		2	7	1
C5.11	(7.81, 37)	(20, 57)	(40.6, 72)		9	16	1
C5.12	(9.70, 37)	(20, 57)	(42.4, 72)		4	16	1
C6.1	(11.37, 37)	(19.8, 57)	(40.8, 72)		5	13	3
C6.2	(8.21, 37)	(14.3, 57)	(42.8, 72)		3	11	1
C6.3	(12, 37)	(28, 57)	(38, 72)		3	0	3

Table 2: Position of corners of piecewise linear function

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C6.4	(17.08, 37)	(28, 57)	(35, 70)		2	4	1
C7.4	(12, 37)	(0, 0)	(42, 72)		3	1	0
C7.5	(19.81, 37)	(30, 57)	(40, 72)		1	1	0
C7.6	(10, 37)	(18, 57)	(31, 72)	0	1	0	
C8.1	(7.81, 37)	(16, 57)	(30, 72)		13	1	0
C8.2	(8.55, 37)	(20, 57)	(35, 72)		11	1	0
C8.3	(9.30, 37)	(20, 57)	(38, 72)		10	19	1
C8.4	(7, 37)	(15, 57)	(34, 72)		6	16	1
SC1	(14.07, 37)	(24.5, 57)	(44,72)		7	29	2
SC2	(12, 37)	(23, 57)	(43, 71)		12	21	3
SC4	(10, 37)	(16, 57)	(40.6, 72)		12	13	2
SC5	(8.38, 37)	(23, 57)	(41.6, 72)		7	10	0
SC6	(10.57, 37)	(18.4, 57)	(36.4, 72)		6	13	2
SC7	(11.77, 37)	(20.5, 57)	(40, 72)		5	8	0
SC9	(11.48, 37)	(20, 57)	(35, 72)		2	4	0

Table 6 on page 8 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Av USM	Rank	Candidates with this USM or above	%
86	1	1	0.99
84	2	3	2.97
82	4	6	5.94
80	7	9	8.91
79	10	14	13.86
78	15	15	14.85
77	16	20	19.8
76	21	23	22.77
75	24	31	30.69
74	32	39	38.61
73	40	45	44.55
72	46	46	45.54
71	47	52	51.49
70	53	57	56.44
69	58	64	63.37
68	65	69	68.32
67	70	75	74.26
66	76	80	79.21
65	81	86	85.15
64	87	89	88.12
63	90	92	91.09

Table 4: Percentile table for overall USMs

Av USM	Rank	Candidates with this USM or above	%
62	93	94	93.07
61	95	95	94.06
60	96	98	97.03
59	99	99	98.02
55	100	100	99.01
48	101	101	100

# B. Equality and Diversity issues and breakdown of the results by gender

Class		Number							
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Ι	8	50	58	6	47	53	11	37	48
II.1	9	31	40	7	19	26	5	18	23
II.2	0	2	2	3	10	13	2	10	12
III	0	1	1	1	0	1	0	1	1
F	0	0	0	0	0	0	0	0	0
Total	17	84	101	17	76	93	18	66	84
Class				Per	rcentag	ge			
		2019			2018			2017	
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Ι	47.06	59.52	57.43	35.29	61.84	56.99	61.11	56.06	57.14
II.1	52.94	36.9	39.6	41.18	25	27.96	27.78	27.27	27.38
II.2	0	2.38	1.98	17.65	13.16	13.98	11.11	15.15	14.29
III	0	1.19	0.99	5.88	0	1.08	0	1.52	1.19
						1	1		
F	0	0	0	0	0	0	0	0	0

Table 6: Breakdown of results by gender

# C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Paper	Number of	Avg	$\operatorname{StDev}$	Avg	StDev
	Candidates	RAW	RAW	USM	USM
C1.1	9	39.44	10.37	76	19.77
C1.2	8	34.38	11.16	71.12	17.14
C1.3	16	36.44	4.99	67.94	7.23
C1.4	12	27.08	7.69	62.33	7.56
C2.1	12	32.33	7.64	74.92	11.07
C2.2	20	38.4	4.89	78.9	8.63
C2.3	4	-	-	-	-
C2.4	10	36.8	10.49	72	16.4
C2.5	8	32.25	5.47	73	8.83
C2.6	10	27.7	7.39	76.4	8.67
C2.7	24	35	5.35	74.71	10.02
C3.1	13	26	8	69.38	10.24
C3.2	10	27.8	8.09	67.5	12.62
C3.3	10	34.3	7.2	72.2	9.93
C3.4	15	39.2	6.5	77.2	8.58
C3.5	10	36.3	9.87	76.2	16.72
C3.7	15	31.73	7.17	71.76	8.75
C3.8	17	33.18	9.1	71.12	9.85
C3.9	4	-	-	74.25	6.98
C3.10	15	23.6	9.42	65.93	17.24
C4.1	11	22.91	7.45	64.64	11.55
C4.3	4	-	-	-	-
C4.4	2	-	-	-	-
C4.6	2	-	-	-	-
C4.8	3	-	-	-	-
C5.1	22	23.73	8.05	62.95	10.16
C5.2	17	28.41	8.66	66.71	12.87
C5.4	28	-	-	69.64	6.98
C5.5	30	31.23	7.05	68.23	6.49
C5.6	21	36.57	9.09	73.57	11.86
C5.7	17	37.82	8.12	70.88	10.57
C5.9	10	28.2	9	64.4	13.77
C5.10	15	-	-	63.86	9.50
C5.11	26	32.96	6.51	66.69	5.78
C5.12	20	33.65	7.42	66	6.91
C6.1	19	32.21	6.88	66.42	6.87
C6.2	14	30.79	8.45	66.43	6.25
C6.3	6	34.67	10.37	69.67	18.01
C6.4	6	39.5	5.15	79	10.33

Table 7: Numbers taking each paper

Paper	Number of	Avg	$\operatorname{StDev}$	Avg	StDev
	Candidates	RAW	RAW	USM	USM
C6.5	11	-	-	68.45	7.32
C7.4	3	-	-	-	-
C7.5	2	-	-	-	-
C7.6	1	-	-	-	-
C8.1	11	27.64	4.97	69.91	5.65
C8.2	9	32.11	7.27	69.89	8.87
C8.3	30	33.97	6.07	69.83	7.18
C8.4	23	28.3	7.52	68.22	8.24
SC1	25	39.88	5.99	70.96	8.37
SC2	16	41.19	6.77	73.75	10.22
SC4	6	34.67	7.2	70.5	9.01
SC5	4	-	-	-	-
SC6	5	-	-	-	-
SC7	1	-	-	-	-
SC9	4	-	-	-	-
SC10	2	-	-	-	-
CCS1	3	-	-	-	-
CCS2	5	-	-	-	-
CCS3	1	-	-	-	-
CCD	62	-	-	74.09	6.96
COD	2	-	-	81	5.65

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

# Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempt		
	All	Used		Used	Unused	
Q1	14.66	17.6	8.98	5	1	
Q2	21.14	21.14	4.18	7	0	
Q3	17.85	19.83	7.55	6	1	

# Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	20.14	20.14	4.33	7	0
Q2	14.14	14.14	6.46	7	0
Q3	17.5	17.5	6.36	2	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attemp		
	All	Used		Used	Unused	
Q1	11.8	14.25	6.37	4	1	
Q2	19.81	19.81	2.94	16	0	
Q3	17.41	17.41	2.67	12	0	

# Paper C1.4: Axiomatic Set Theory

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	14.41	14.41	4.14	12	0
Q2	12.66	12.66	6.06	12	0
Q3	-	-	-	-	-

# Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	18.54	18.54	4.45	11	0
Q2	13.83	13.83	3.78	12	0
Q3	18	18	-	1	0

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	19.06	19.06	2.76	15	0
Q2	19.11	19.11	2.71	17	0
Q3	19.62	19.62	4.24	8	0

Paper C2.3: Representation Theory of Semisimple Lie Algebras

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	23	23	2.82	2	0
Q2	16	16	-	1	0
Q3	15.5	15.5	1	4	0

# Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	18.6	18.6	6.31	10	0
Q2	16.25	16.25	6.34	4	0
Q3	19.5	19.5	3.20	6	0

# Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16.25	16.25	1.83	8	0
Q2	14.85	14.85	3.89	7	0
Q3	24	24	-	1	0

# Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16.7	16.7	4.00	10	0
Q2	8.5	8.42	3.07	7	1
Q3	17	17	5.19	3	0

# Paper C2.7: Category Theory

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	15.95	16.68	3.68	19	2
Q2	16.35	16.35	3.20	14	0
Q3	19.6	19.6	3.50	15	2

# Paper C3.1: Algebraic Topology

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	15.6	15.6	6.63	10	0
Q2	10.57	10.57	4.03	7	0
Q3	12	12	2.29	9	0

# Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16	17.33	5.84	9	1
Q2	11.16	11.16	7.27	6	0
Q3	8.85	11	4.22	5	2

Paper C3.3: Differentiable Manifol	$\mathbf{ds}$
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Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	18.55	18.55	3.08	9	0
Q2	17.33	17.33	5.56	9	0
Q3	10	10	2.58	2	2

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	19.30	19.30	2.65	13	0
Q2	19.66	19.66	7.28	6	0
Q3	19.25	19.90	3.30	11	1

# Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	17.12	17.12	4.48	8	0
Q2	18.84	18.84	6.73	10	0
Q3	14	21	9.05	2	2

# Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	14.30	14.30	2.78	13	0
Q2	14.55	16.24	5.63	7	2
Q3	17.5	17.5	5.70	10	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	20.75	20.75	2.87	4	0
Q2	14.8	15.5	6.03	14	1
Q3	16.5	16.5	4	16	0

# Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	9	9.81	6.39	11	1
Q2	11.28	11.83	3.94	6	1
Q3	12.64	13.46	6.23	13	1

# Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	8	8.85	4.27	7	1
Q2	13.55	13.55	4.50	9	0
Q3	9.33	11.33	4.71	6	3

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	18	18	3.91	4	0
Q2	14.75	14.75	0.95	4	0

# Paper C4.4: Hyperbolic Equations

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	6.5	6.5	3.53	2	0
Q2	8.5	8.5	0.70	2	0
Q3	4	-	-	0	1

Paper	C4.6:	Fixed	Point	Methods	for	Nonlinear	PDEs

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	12.5	12.5	4.94	2	0
Q2	15.5	15.5	0.70	2	0

# Paper C4.8: Complex Analysis: Conformal Maps and Geometry

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	10.33	10.33	1.15	3	0
Q2	15	15	1.41	2	0
Q3	17	17	-	1	0

# Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	11.90	12.14	4.68	21	1
Q2	5.44	6.33	2.69	3	6
Q3	12.4	12.4	4.91	20	0

# Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	8.62	8.62	5.09	8	0
Q2	16.76	16.76	3.49	13	0
Q3	14.14	15.07	6.06	13	1

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	17.25	17.25	5.35	27	0
Q2	14.84	14.84	3.58	26	0
Q3	11.87	12.14	2.47	7	1

# Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.87	18.87	4.17	16	0
Q2	13.26	16.09	7.30	11	4
Q3	19.26	19.26	5.36	15	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	17.4	17.4	4.85	15	0
Q2	20.17	20.17	3.94	17	0
Q3	13.33	19.5	11.59	2	1

# Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	18.83	18.83	2.56	6	0
Q2	9.6	9.6	4.08	10	0
Q3	14.8	18.25	11.73	4	1

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16.72	16.72	3.39	22	0
Q2	14.95	15.33	4.61	21	1
Q3	17.7	18.55	3.94	9	1

# Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16.37	16.37	4.27	8	0
Q2	16.88	16.88	3.89	18	0
Q3	17	18.30	5.96	13	1

# Paper C6.1: Numerical Linear Algebra

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	13.11	13.31	4.34	16	1
Q2	18.07	18.07	3.14	14	0
Q3	16.88	18.25	5.15	8	1

# Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16	16	3.58	13	0
Q2	15.5	17.71	7.55	7	1
Q3	12.37	12.37	5.97	8	0

# Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	17.83	17.83	5.26	6	0
Q2	20.66	20.66	4.16	3	0
Q3	13	13	4.35	3	0

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	13	20	8.28	2	2
Q2	21.5	21.5	3.50	6	0
Q3	15.6	17	5.77	4	1

Paper C7.4: Introduction to Quantum Information

Question	Mean Mark		Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	23	23	-	1	0
Q2	21	20	5.29	2	1
Q3	23	23	2.64	3	0

# Paper C7.5: General Relativity I

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	18	18		1	0
Q2	20.5	20.5	3.53	2	0
Q3	18	18		1	0

# Paper C7.6: Relativity II

Question	Mea	an Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q2	22	22	-	1	0
Q3	14	14	-	1	0

# Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	16.18	16.18	3.91	11	0
Q2	11.33	12.12	3.42	8	1
Q3	8.4	9.66	2.30	3	2

# Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	17.77	17.77	2.68	9	0
Q2	14.12	14.12	5.86	8	0
Q3	16	16	-	1	0

# Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	17.07	17.11	3.49	26	1
Q2	16.87	16.87	2.96	16	0
Q3	16.88	16.88	4.01	18	0

### Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	11.90	11.90	5.20	21	0
Q2	16.63	16.63	3.68	22	0
Q3	10.5	11.66	4.50	3	1

# Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16	5.90	14	0
Q2	22.60	22.60	1.64	23	0
Q3	19.46	19.46	2.66	13	0

Question	Mean	Mark	Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	19.57	19.57	3.52	14	0
Q2	16.12	17.28	5.64	7	1
Q3	24	24	0.89	11	0

# Paper SC2: Probability and Statistics for Network Analysis

# Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempt	
	All	Used		Used	Unused
Q1	17.16	17.16	5.49	6	0
Q3	17.5	17.5	2.88	6	0

# Paper SC5: Advanced Simulation Methods

Question	Mean Mark		Std Dev	Numb	per of attempts
	All	Used		Used	Unused
Q1	17	17	7.74	4	0
Q2	19.25	19.25	3.86	4	0

# Paper SC6: Graphical Models

Question	Mean Mark		Std Dev	Number of attemp	
	All	Used		Used	Unused
Q1	12.75	14	3.30	3	1
Q2	17	17	4.39	4	0
Q3	12.66	12.66	3.05	3	0

# Paper SC7: Bayes Methods

: Bayes Methods										
	Question	Mean Mark		Std Dev	Number of attempts					
		All	Used		Used	Unused				
	Q1	11	11	-	1	0				
	Q2	18	18	-	1	0				
	Q3	5	-	-	0	1				

# Paper SC9: Interacting Particle Systems

Question	Mear	n Mark	Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	0.81	4	0
Q2	14.5	14.5	3.10	4	0
Q3	6	-	-	0	1

# Paper SC10: Algorithmic Foundations of Learning

Question	Mear	n Mark	Std Dev	Number of attempts		
	All	Used		Used	Unused	
Q1	18.5	18.5	0.70	2	0	
Q2	24	24	0	2	0	

# D. Recommendations for Next Year's Examiners and Teaching Committee

None

# E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

# C1.1: Model Theory

The three questions were chosen about evenly.

1) There was a bit of a dichotomy here. About half clearly saw the need for the upwards Loewenheim-Skolem theorem in (a), for the the preservation of universal sentences under substructures in (b) and of existential sentences under going to superstructures in (c), as well as the fact that a complete theory of a finite structure determines a cardinality. These answers were generally good. Several did not attempt part (c) or even part (b), which covers basic material around the definition of a structure.

2) Generally people did quite well, showing a clear understanding of the model theory. Nor did the simple algebra faze anyone; most replies constructed isomorphisms for models of  $\alpha$ and  $\beta$  and then putting them together in the case of C. In several cases the construction was unnecessarily elaborate (repeating back and forth constructions not needed here), but no points were taken off for this. Many lost one point for a slightly incorrect statement of the Los-Vaught test. A number did not correctly give the theory of C, for instance asserting that  $\sigma$  along with the negations of  $\alpha, \beta$  suffices, not realising that each possibility must be asserted to occur infinitely often.

3) Most stated the omitting types theorem correctly, and showed a good understanding in parts (b) and (c). Part (d) gave more trouble; many realised that an  $\aleph_1$ -categorical but not  $\aleph_0$ -categorical is needed and explained why, but could not think of one.

#### C1.2: Gödel's Incompleteness Theorems

Among the weaker candidates, a few used the wrong definition of what it is for a set to be expressed by a formula, or for a set to be enumerated by a formula in a formal system. Some candidates lost a mark by using without argument a result from the course not given in the preamble or allowed by being stated clearly in 2(c) and 3(c), for example citing  $P_6$ , where what's given, by (iii), is that  $Pr(v_1)$  is a provability predicate for PA (needed to say something like: " $P_4$  by  $P_1$  and  $P_2$ ,  $P_5$  by  $P_4$  and  $P_2$ , and  $P_6$  by  $P_5$  and  $P_3$ ").

Q1 is on arithmetization of syntax and the first incompleteness theorem. In part (a) the weaker candidates did not discuss the fact that  $Tm(v_1)$  and  $Fm(v_1)$  are  $\Sigma_1$ , and how, despite this fact,  $\{n : PA \vdash E_n \text{ is expressible by a } \Sigma_1\text{-formula, and omitted to say anything about the role played by the arithmetization of <math>v_2 \prec v_3$ . Part (b) called for a proof of the second half of the First Incompleteness Theorem, so it was bookwork, but with the twist that it was to be proved from the provability of the diagonal equivalence, while in the lectures it was proved from the expressibility of  $\{n : PA \vdash E_n[\overline{n}]\}$ , and one candidate on autopilot gave that proof. Part (c) covered the first half of the First Incompleteness Theorem, though with the twist of going directly to  $\omega$ -incompleteness. Several candidates streamlined the bookwork by going directly from the supposition that  $PA \vdash G$  to  $PA \vdash Pr(\overline{\lceil G \rceil})$  since it was given  $P(v_1)$  is a provability predicate (which had not been established yet when that result was proved in the lectures). Part (c) was a new result, but made very manageable by the hint, and candidates who weren't frightened off by not having seen this result before did well with it.

Q2 is on the Second Incompleteness Theorem and the background to Rosser's Theorem, though not on Rosser's Theorem itself. There were some excellent answers, but overall candidates did markedly less well on this question than on Q1, despite the fact that this question was very close to bookwork, with an average mark of 13.6. Part (a) called for a proof of the Second Incompleteness Theorem for PA, i.e. for any sentence X,  $PA \nvDash$  $\sim Pr(\overline{\lceil X \rceil})$ , in the equivalent form  $PA \cup \{Pr(\overline{\lceil X \rceil})\}$  is consistent, a form of equivalence by propositional logic much used in the course (as well as in proving the Completeness Theorem for First-Order Logic, a prerequisite to the course). Part (b) was problem 2(c) on Problem sheet 5, and received a number of excellent answers, but a number of candidates struggled with it, some because they weren't working with the right definition of what it is for a formula to express a set. Part (c) is an immediate consequence of the Separation Lemma, which a number of candidates realized, but some didn't (despite the fact that this fact entered into the solution of problem 2(d) on Problem sheet 5). Part (d) is not bookwork. Some candidates made progress, while a number got nowhere. It follows from propositional logic by taking  $Pr_{PA\cup\{Pr(\overline{\lceil 0=0'\rceil})\}}(v_1)$  to be  $Pr(\overline{(\lceil Pr(\overline{\lceil 0=0'\rceil}) \supset \rceil} * v_1 * \overline{\lceil})^{\neg}),$ which is justified by the Deduction Theorem for first-order logic and given (iii). One mistake which several candidates made was to take it that  $Pr(v_1)$  is a proof predicate for S. Some candidates included claims that contradict the Second Incompleteness Theorem.

Q3 is on Löb's Theorem and the Fixed Point Theorem for Provability Logic. Part (a) called for a proof of Löb's Theorem, in the context of determining the truth or falsity the Henkin sentence ("this sentence is provable"), which is how Löb's Theorem was introduced in the lectures (and historically). Part (b) tested the point, stressed in the lectures, that the inference from  $S \vdash A$  to  $S \vdash B$ , as in the case of Löb's Theorem, does not thereby imply  $S \vdash (A \supset B)$ . Part (c) is straight bookwork; the candidates who got this question

expounded proofs that were well expressed in their own way, i.e. not memorized from the proof given in the lectures. Part (d) called for verification of a fixed point, which could be done in various ways. There were some good solutions, which were by the method of solving for a fixed point of the result of substituting a decomposition into a component, and proving that instance of the Fixed Point Theorem; the third solution was a two-line proof from the result in (c)(a third possibility is by direct derivation in GL).

#### C1.3: Analytic Topology

q1: This question was not popular, though there were some good solutions of it.

The most common errors in part (a), which was bookwork, were in the proof of second countability from one of the other conditions: either a countable family of open sets was defined such that every non-empty open set contained a member of this family, but was not actually a basis; or a candidate basis was defined, but confusion involving the triangle inequality meant that given a point x in an open set U, the basis element chosen was too big by a factor of 2, and was not guaranteed to be contained in U.

There were many ingenious solutions to (b)(i), though the most obvious solution— $\rho(x, y) = \min(d(x, y), \epsilon)$ —did not occur to most people. Many had the right idea for (b)(ii). It should be noted that although candidates were not required to demonstrate that their candidate metric was a metric, they were required to show that it generated the product topology.

Many failed to spot the solution to (c)(ii): namely, that A needs to be countable; and of these many failed to spot that (iii) is now trivial, for a space that is Lindelöf but not separable cannot be metrisable by part (a). Perhaps this was due merely to shortness of time.

q2: This question was attempted by all candidates.

Part (a) was generally well done.

Most people did (b)(i) and (ii) well. There were good attempts also at (iii); the most common error was to fail to show that the basis of clopen sets exhibited, actually generated the Tychonoff topology rather than some other.

All that is required for part (c) is to apply Stone duality to the previous parts of the question. Quite a few failed to spot this (perhaps due to shortness of time), including some that clearly understood category theory very well and knew that what part (c) was about was defining a coproduct operator in the category of Boolean algebras.

q3: This was quite popular.

Part (a) was on the whole done well, though some people produced functions f for part (ii) that were not continuous. One candidate produced a solution for part (v) which was slick, economical, and did not use Urysohn's Lemma: namely  $g = (D_A - D_B)/(D_A + D_B)$  (where A and B are respectively the odd and even members of the sequence of  $y_n$ ).

There were some good solutions to (b)(i) and (ii), though some proofs of (iii) were defective in various ways.

(b)(iv) caused significant difficulty and there were few correct solutions. One direction is trivial (if X is compact then it is homeomorphic to  $\beta X$ , so if X is also metrisable then so is  $\beta X$ ). The other direction uses (a)(v) to show that no sequence on X can converge to a

point of  $\beta X$  not in the range of the embedding of X in  $\beta X$ , so that  $\beta X$  can then not be metrisable. One common error was to assert that X is always closed in  $\beta X$  (true if and only if X is compact).

### C1.4: Axiomatic Set Theory

Almost every candidate attempted questions 1 and 2 and the standard of answers was generally high, with maybe a slight lack of accuracy hurting candidates.

**Question 1** In part (b), for the requested example, a lot of correct classes but the justifications were often somewhat incorrect: sometimes it was not ensured that  $A \subseteq B$  although absoluteness was only defined in this context; much more often one of the x and z did not belong to A so that again the definition of absoluteness would not apply;

Only few candidates managed to make progress on (d) claiming erroneously that a formula with  $\forall x \in \mathcal{P}(y) \dots$  would be  $\Delta_0$  (and sometimes they did so in (c) as well) or absolute as long as A, B satisfy **Powerset**. However both a direct proof and a proof using the fact that (x, r) is well-ordered if and only if it is order-isomorphic to an ordinal (with  $\in$ ) were given.

**Question 2** In part (a), candidates were not always clear which facts they used (and thus had to prove) and in particular where transitivity of the  $V_{\alpha}$  and V is needed.

In part (b), candidates usually obtained full marks or very few marks with a wide variety of mistakes e.g. choosing a  $y_x$  for each x or assuming that  $\{y : \phi(a_1, \ldots, a_n, x, y)\}$  was a set.

In part (c), again some very nice answers were given but the majority of candidates 'guessed' the wrong formula  $x \notin y \wedge x = \{x\}$ .

# C2.1: Lie Algebras

Almost all students attempted question 1 and 2.

Question 1: Many good solutions, thought surprisingly few candidates found the correct approach to (d) using Lie's theorem to the solvable Lie algebra spanned by X and Y.

Question 2: Good solutions. For the last part most candidates guessed that any derivation of  $\mathfrak{g}$  must send the ideals  $\mathfrak{b}$  and  $\mathfrak{h}$  to themselves, but few managed to prove it.

Question 3: Only one attempt on this question.

# C2.2: Homological Algebra

The paper was of adequate difficulty, and there were no questions that I deemed either too easy or too hard.

# C2.3: Representation Theory of Semisimple Lie Algebras

No report.

# C2.4: Infinite Groups

**Question 1** This question was attempted by all candidates, with the largest number of correct answers. Some candidates did not use an induction on the class of nilpotency. A few did not understand that the main point of the last question was to show that the torsion was a group. About half did not see that an important part of the argument was that the torsion, when a subgroup, is characteristic, hence the torsion of a normal subgroup is itself normal.

**Question 2** This question was attempted by the least number of candidates. Possibly due to the fact that residual finiteness is a less familiar notion. The second part, on the residual finiteness of a rather simple wreath product, was not attempted by many, and despite the fact that the questions almost led to the solution, the final answer has been provided by very few candidates.

**Question 3** This question was the second in terms of popularity. While the first and the third part were well answered, in the second most candidates were able to prove that the homomorphism from the quotient of the free group to the dihedral group is onto, but failed to prove that it is one to one.

#### C2.5: Non-Commutative Rings

Question 1: This was a very popular question with good results. Part (d) was the hardest. A common error was to try to use determinants, which only works in commutative rings. No candidate spotted that it was proved in a problem sheet that a regular element of an Artin ring is a unit, and of course  $M_n(D)$  is an Artin ring.

Question 2: Another very popular question. Part (c) was a common difficulty with many incorrect applications of Jacobson's theorem. Surprisingly few candidates managed to answer (d) correctly.

Question 3: Few attempts but with good solutions.

### C2.6 Introduction to Schemes

Q1 and Q2 were by far the most popular.

Q1 (c) Very few students gave a complete answer. The point is that f gives rise to a morphism of rings  $\phi : \mathbf{C}[x] \to \mathbf{C}[x]$  and an  $\alpha \in \mathbf{C}$  such that the element  $\phi(x - \alpha)$  is not contained in any prime ideal. Hence  $\phi(x - \alpha)$  is a non zero constant, from which the result follows.

Q2 (b) Very few students gave a complete answer. One way to proceed is to show that each point of S corresponds to an affine open subscheme and to use this fact to show that the global sections functor on the category of quasi-coherent sheaves on S is exact. The result then follows from Serre's cohomological criterion of affineness.

Q2 (c) Very few students gave a complete answer. One way to proceed is to show that the direct image functor  $f_*$  from quasi-coherent sheaves on S to quasi-coherent sheaves on T is exact. This may be seen from a computation on stalks (noticing that the stalks vanish outside the image of f). One may then suppose that T is affine and apply Serre's cohomological criterion of affineness.

Q3 Was attempted by only a few students.

Q3 (b) (ii) Many students gave a correct answer, but very few justified their answer properly.

Q3 (c) (ii) Follows from Q3 (c) (i), because if  $H^1(S, \mathcal{I}) = 0$  then  $H^0(C, \mathcal{O}_C)$  is a field, which is a contradiction.

# C2.7 Category Theory

Question 1 was the most popular question, with some very good answers though no perfect solutions. The other two questions attracted equal numbers of attempts; question 3 was found easier by candidates who had mastered the material in the later part of the course. It was pleasing that nearly all candidates were able to give good answers to at least the more straightforward parts of two questions, but disappointing that almost nobody gave a correct description of co-equalisers in the category of sets.

#### C3.1: Algebraic Topology

Question 1: (b) (i) Note that  $\mathbb{CP}^2$  is a manifold; no geometric computation is required. (ii) The result follows from the Künneth theorem. (c) (i) There is no ring homomorphism that reverses sign in top degree, therefore no orientation-reversing self-homeomorphism.

Question 2: (a) Note that specifying the degree of the attaching maps does not determine the maps and therefore does not specify a CW structure. (b) (i) Though possible to compute the relative cohomology using a long exact sequence, it is more direct and in this case reliable to compute directly from the relative cell complex. (ii) Computing the cohomology directly from a cell complex makes clear the generators of the cohomology groups, which aids intuition in the following part. (c) (i) As in question 1, no geometric computation is required, only induction using the manifold structure. (ii) This challenging part is best approached by considering the reduction mod 2 map, with careful attention to which maps are and are not isomorphisms, informed by part (b).

Question 3: (b) (i) The cohomology groups could be those of a manifold, but the ring structure cannot be that of a manifold. (ii) Mayer–Vietoris determines the cohomology groups of the connect sum. (c) (ii) Recalling the graded commutativity of the cohomology ring helps nail down the ring structure.

#### C3.2 Geometric Group Theory

**Q1** This was a basic question about residually finite groups, presentations and algorithmic problems. All students attempted this. The substantial part of 1.a was the proof that residually finite groups are Hopf and most students gained full marks on this.

Parts b i, ii were generally well done. In part iii some candidates gave a very vague description of the enumeration procedure and marks were taken off. Part iv was the most challenging and most students made some mistakes when explaining the procedures that run in parallel. Some assumed that the word problem is solvable while others did not explain how they check that a homomorphism is onto. Part v was generally well done.

**Q2** This was a question on amalgamated products and actions on trees. In part a some students gave the definition of the fundamental group rather than a presentation as it was required. In part b most candidates did well in the first part but several were confused in the last part trying to use for example the Cayley graph instead of the quotient graph of groups provided by the action.

Part c was generally done well but a common mistake was to assume that if there is an edge labelled by  $\mathbb{Z}$  in the quotient graph of groups then the group splits over  $\mathbb{Z}$  (i.e. they did not rule out trivial splittings where both an edge and a vertex are labelled by the same group).

**Q3** This question was on the last part of the course dealing with quasi-isometries and hyperbolic groups.

Part a.i was well done. In part ii many students used the correct diagram but failed to give a complete argument using thin triangles. Many students saw how to apply a.ii and gave a complete solution of b.i. Some students managed to do b.ii and some found u, v but did not prove the inequality and got partial credit. Nobody managed to do b.iii.

#### C3.3: Differentiable Manifolds

Question 1: Attempted by most candidates. Part (c) had quite a lot of things to cover and even candidates who knew what they were doing tended to lose a few marks by missing things out (e.g. by not explaining why X is Hausdorff, and second countable, which is why f-1(y) was supposed finite or countable).

Part (d) was done poorly. The answer is that f is not a covering map (because of behaviour over 1 in Y as 0 is not in X), but it is a local diffeomorphism. You could have inferred the first from the question just on logical grounds: as (c) shows that covering maps are local diffeomorphisms, if f were a covering map, then there would be no point in the examiners also asking if f is a local diffeomorphism. But almost everyone said f is a covering map, and a surprising number did not answer the question about local diffeomorphisms.

Question 2: (a),(b) were bookwork and done well. Candidates found the first parts of (c),(d) difficult ((c)(i) needed an algebraic trick which most did not spot, though many got part marks; in (d) I was disappointed by how few could explain that  $S^1$ -invariant k-forms  $\alpha$  on  $S^1 \ge Y$  are  $d \ge \wedge \beta + \gamma$  for  $\beta, \gamma k - 1, k$ -forms on Y), but the second parts of both were easy marks for those that kept their heads.

Question 3: The least popular question. For (a), some candidates did not know the definition of orientations on manifolds in terms of orientations on tangent spaces. Part (d) was difficult, and candidates who gave up and did not attempt it received at most 13 marks.

#### C3.4: Algebraic Geometry

Almost all candidates chose exercise 1, after which as second option exercise 3 was about twice as popular as exercise 2.

Exercise 1: (b) almost all candidates did not take the closures of the C-sets, confusing the condition of being relatively closed in the qpv X with being closed in the ambient projective space; (c) surjectivity seems to have stumped many candidates even though it was clear due to there being a quotient map on coordinate rings for subvarieties; (d) frequent mistake: candidates used isomorphisms f,g to identify the qpvs V,W with affine varieties A,B, and then took A intersect B, but A intersect B is in general unrelated to V intersect W. Only very few candidates used the map (fxg) applied to ((VxW) intersect (Diagonal)), and the fact that (fxg)(Diagonal) is closed in AxB.

Exercise 2: (a) candidates often wrote the definition of tangent space for an affine variety in terms of a vanishing set, rather than the intrinsic definition needed for a projective variety or a qpv; (d) candidates sometimes did not see that one had to consider the Pluecker embedding, in order to justify why the map was a morphism.

Exercise 3: (a) many candidates did not explain to which algebra g,h belong, when writing f = g/h, in the definition of regular function; (b) a lot of confusion by candidates caused by using the coordinates  $x_j$  (with  $x_i$  omitted), rather than  $x_j/x_i$  on the affine charts  $U_i = (x_i \text{ not zero})$ . Candidates erroneously thought that the function was therefore a polynomial in the  $x_j$  on  $U_i$  independent of  $x_i$ , and therefore the polynomial was independent of all coordinates  $x_i$ , hence constant! (c) Some candidates stated what algebras are involved, but without saying how the equivalence maps objects and morphisms; (d) most candidates forgot that the Veronese embedding allows one to prove that  $P^n(F)$  is affine (proved in the notes, and arises in a homework exercise).

#### C3.5: Lie Groups

Question 1 This question was about Lie subgroups and subalgebras, with some links to representations and maximal tori.

Most candidates were fine with the subgroup and subalgebra concepts, but some found describing the decomposition of representations more tricky.

The last part (finding disconnected Abelian subgroup not contained in a maximal torus) proved harder, as expected, but several candidates managed this successfully.

Question 2 This question was on representations of SU(2) and characters. This question proved very popular, with most candidates producing good solutions. Most candidates were fairly expert with characters and how to apply them, though not all were able to put all the pieces together to get the quick proof of the final part (many did manage this, however).

Question 3 This question, on adjoint maps, Killing form and decomposition into root spaces, was the least popular. The bookwork on Ad and ad was generally well done, and most candidates were fine with the Killy form calculations. The final part required some insight into root spaces and proved more challenging.

#### C3.7: Elliptic Curves

Question 1:

Part (a) was done quite well, although many candidates were happy to apply the Hasse estimate directly at primes dividing the discriminant in (iii), and others laboured over determining the group structure in (ii) rather than just observing there are only two groups of order 4 and one can be easily ruled out. Part (b)(i) was done well, though sometimes the methods used were lengthy. Only one student made progress in (b), spotting the connection between the expression and the result of repeatedly applying the operator  $\frac{x}{d}dx$  to the power series for 1/(1-x).

#### Question 2:

The bookwork in parts (a) and (b) was done well. Part (c) was original and harder, but a number of students did this well, even completing it.

#### Question 3:

The marks on this question were overall higher than the others, with several candidates getting near full marks or full marks. Most students had a good idea what to do, as similar questions will have been encountered in past examinations.

### C3.8 Analytic Number Theory

Overall the exam seemed well-balanced, with a good spread of marks.

Question 1 This question was much less popular than the other two, although candidates who did attempt it tended to do well. This is presumably because a large part was based on bookwork of one of the harder parts of the course. There were slight mishaps with the final rearrangement of terms in part (c) and balancing the error terms in part (d), but generally most parts were competently answered.

Question 2 This question was popular, with a fairly wide spread of marks awarded. More candidates had difficulty with part (a) than expected, but part (b) was almost uniformly well-answered. Candidates who saw the main idea to swap the order of summation in (c) generally answered well, but occasionally found difficulties with adequately handling error terms. No candidate gave a perfect answer (showing analyticity) to part (d), but several saw the main idea to separate the main term inside the partial summation step.

Question 3 This question was also popular, being attempted by almost all candidates. The first three parts of the question were easier and generally correctly answered. Several candidates had rather more difficulty answering part (d). Most were aware of the basic strategy which should be adopted for (d)(i), but had difficulty adequately bounding each error term. Part (d)(ii) was quite well answered, even by candidates who struggled with the earlier part of the question.

#### C3.10: Additive and Combinatorial Number Theory

Q1. This question was on a part of the course that many students found hard, so I kept to the basics. Even then, the attempts were quite poor, even at the bookwork. There were no satisfactory attempts at part (b), which really only requires a small modification to the proof of (a). Part (c) saw quite a few solutions, but also a good number of completely wrong solutions, which is disappointing since consequences of the orthogonality relations such as this were ubiquitous in the course and on the exercise sheets. Finally, very few candidates made a correct attempt at (e), assuming that the bound they had stated for Gauss sums in (c) applied without the restriction that q be coprime to a.

Q2. Attempts here were quite poor with the exception that quite a few students did well on part (b). I was particularly disappointed that no student managed either (a) (ii) or (a) (iii).

Q3. Subdivided marks have been annotated on the official solution. Unsurprisingly, this was the most popular question, being on the second half of the course which the students generally found easier. The bookwork was mostly well done and there were some easy marks available for that. However, as soon as the question veered away from bookwork, candidates found trouble. No candidate managed to construct (greedily) a subset of  $1, ..., n^3$  with no

additive relations. And no candidate properly justified lifting from Z/qZto[q] so as to apply Roth's theorem, in the last part.

#### C4.1: Further Functional Analysis

There were some strong performances, but overall the cohort appeared to be rather less well prepared than others in recent years.

**Question 1.** The general standard of the answers to this question was low. In part (a) some candidates stated the wrong version of the Hahn-Banach Theorem, and a surprising number struggled with part (ii). Part (b) was a minor variation of an exercise discussed in one of the classes, but disappointingly there was not a single complete solution. Few candidates made serious attempts at part (c), and those who did tended not to see how the existence of Banach limits established in (b) could be used to answer part (ii).

Question 2. The bookwork in part (a) was on the whole handled competently. Part (b) had appeared as a question on a problem sheet, and also on a recent past paper, but even so it was pleasing to see how many of the candidates were comfortable with the relatively intricate argument required here. Part (c)(i) received several good answers, but only one candidate was able to see how exactly the result from part (b) could be used in part (c)(ii).

**Question 3.** Parts (a) and (b) attracted a number of high-quality answers, although some candidates, alarmingly, came a cropper even in these early stages. Part (c) offered an alternative approach, mentioned in passing during the lectures, to the spectral theory of compact operators. The results were mixed. In part (d) candidates were generally able to see how to make use of the FTC but nobody managed to produce a complete solution.

#### C4.3: Functional Analytic Methods for PDEs

The first question has been attacked by all candidates. In (b), many candidates do very well, applying the extension scheme and Poincare-Sobolev inequality in the last stage. The first part of (c) is based on the interpolation in  $L_p$ -spaces and the Gagliardo-Nireberg inequality. It has been noticed by most of candidates. The second part of (c) is just technical and can be done with the help of Young inequality.

Many students did the second question. In (b), the embedding of  $W^{1,n}$  into BMO is proved easily if Poincare-Sobolev and then Hölder inequalities are applied. The most difficult part of (c) is show that logarithm is in BMO. Nobody can finish (d) although the idea is the same as in the proof of Hölder continuity: one needs to compare the mean value over balls of a given radius with the mean value over balls of the double radius.

The third question was not popular among the candidates but was done very well by those who did take this question.

#### C4.4 Hyperbolic Equations

Question 1: Part a) of the question was answered correctly by nearly everyone, however the bookwork parts b) and c) posed considerable difficulties for the candidates. The construction of the unique entropy solution in part d) was attempted by all the candidates, but everyone made a mistake at different stages and none succeeded in finding the correct equation for the first shock curve. Some candidates got very caught up in wrong computations and lost a lot of time here which was clearly lacking for the later questions.

- Question 2: All candidates struggled with the proof of the one dimensional Sobolev embedding in part a)(i) of the question and the energy estimate in part a)(ii). Part b) was received well; the candidates had in general a correct approach to the solution however some did not make use of the a priori boundedness of the solution to conclude the argument.
- Question 3: Part a)(i) of the question did not pose any difficulties, the other parts were not attempted probably because of lack of time.

#### C4.6 Fixed Point Methods for Nonlinear PDEs

Question 1: Part a) was very close to what was done during the lecture. Half of the students answered it correctly, while the other half wrote the pieces but did not manage to combine them in the correct way or to develop them fully. In part b) some students also struggled. There were many ways to answer it, yet most attempted one done during the lecture. Unfortunately, this was longer than the alternatives (and therefore, had more places where students could make mistakes and took more time that could have been useful later). Part c) seemed to be understood conceptually, but it had some technical differences to things done during the lecture. Many students failed to realize and tackle those particularities in the right way. Only one student noticed that C was unbounded, and then still tried to use the wrong version of Schauder's theorem.

**Question 2:** Part a) was very easy, yet some students either forgot the dependency of f on u, or got confused with the Sobolev spaces. Part b) was received well, although most of students took the long way to prove compactness, instead to appeal to the results we studied in the lecture. In doing so, they missed some important details. For example, they used Dominated Convergence theorem, without checking all assumptions.

In part c)(i), many students struggled to use the weak maximum principle. Part c)(ii) was full of small computation mistakes that might indicate that although students understand the overall concepts, they needed more practice with this type of calculations. Part c)(iii) Was straight forward and in general students had a clear idea of what to do. However, most of them appealed to the compactness shown in part b) without noticing that f had different properties in this context.

**Question 3:** This was the less attempted question. Part a) was bookwork and well received. In part b) none of the students did what the sample solution suggested and used integration by parts to pass the derivatives to v. nevertheless, they applied all arguments correctly to justify their choice. It should be noted that there were again some minor computation mistakes. Part c) was wrongly answered by students, who wrote a functional depending on u and v (instead of only on u). All students struggled with part d). Part e) showed again that students lacked practice with norms and bound computations. Nevertheless, in general they identified what to do.

### C4.8 Complex Analysis: Conformal Maps and Geometry

Q1: This is by far the most popular question attempted by all candidates. Part (b)(i) turned out to be very hard despite being a minor modification of a problem from the first problem sheet. Very few students completed part (c) which is very similar to the proof of the Riemann mapping theorem.

Q2: In part (b) all students failed to realize that the family of the curves is not symmetric with respect to re-labelling and the problem is *not* the same as the one from the problem sheet. No one noticed that the problems in parts (b)(i) and (b)(ii) are equivalent to each other.

Q3: This is the easiest question, the main difficulty was in parts (c)(ii)-(iii) where one was supposed to use the growth theorem.

### C5.1: Solid Mechanics

Q1: All students tried this question but only a few manage to do it well. The question was mostly technical and many students had difficulties with algebraic manipulation. The first 15 marks were mostly straightforward matrix products. Some students showed a decent understanding on the basics of nonlinear elasticity. Few students manage to complete the last part that required a better understanding of the material.

Q2: This question was probably the hardest in terms of theoretical concepts but easy in terms of manipulation. Unfortunately, none of the students went further than the first few steps. It was disappointing to see that many students could not give the correct physical dimensions of the quantities appearing in the Cauchy equation (an easy 5 marks).

Q3: Similarly to the first question, almost all students tried this question. Again, many students had difficulties with tensorial manipulation. A few students showed good understanding of the material and could easily obtain the answer (an easy 3 marks).

#### C5.2: Elasticity and Plasticity

Question 1 This was the least popular question and was generally not answered well. Several students erroneously set the displacement to zero at x = 0 instead of the stress, despite the required boundary conditions being clearly set out in part (a). Most candidates were able to derive the relations for the reflection angles in part (b), but almost none gave a convincing argument for the form of  $\beta$  when  $\sin \alpha > c_s/c_p$ . In part (c), a combination of incorrect boundary conditions and faulty algebra defeated almost everyone, and only one candidate made any headway whatsoever with part (d).

Question 2 This question was relatively popular and attracted several good solutions. The bookwork in parts (a) and (b) was generally handled well, although there was some sloppy manipulation of inequalities. The example in part (c) was similar to problems done in lectures, and most who attempted it got the required equation relating  $\delta$  and s. Almost no-one convincingly completed part (d), which required a bit of thought about the mutual reaction force between the two strings.

**Question 3** This was the most popular question, and the attempts covered a wide spread of marks. Most candidates managed the bookwork in part (a), albeit sometimes laboriously and with some uncertainty about the direction of the inequality. In part (b), very few realised that the identity from (i) can be used to simplify the following calculations. Several candidates were confused about the sign convention for the pressure (for example setting  $\tau_{rr} = +P_{in}$  at r = a), and there was also some ambiguity about the sign of the square root in the Coulomb condition. In (iii), very few candidates successfully formulated a differential equation for the stress and thus solved for s.

#### C5.5: Perturbation Methods

**Q1** Overall this was very popular and implemented well in the earlier stages. However, errors did accumulate over the course of the question in many solutions and the Van Dyke matching challenged every candidate. In particular, while the best answers clearly understood the principles and were very close to a perfect solution, errors generally prevented a perfect score.

Q2 This question was also popular and the first part was implemented very well. The steepest descent part of the question was found to be more challenging than perhaps expected though there were a number of very high scores. In a number of cases with lower scores and extensive answers to Q1, it appeared that time may have been a factor. Nonetheless, not observing the final integral could be written in terms of a complex exponential in the integrand did increase the work of a number of students and a failure to evaluate only the contour integrals required for the final answer also limited progression in the question.

**Q3** This was not a popular question. When attempted, most attempts gathered all marks in the earlier parts. The latter parts were found to be difficult though the better answers clearly saw how to conceptually tackle the problem, even if slips prevented full solutions.

# C5.6: Applied Complex Variables

No report.

#### C5.7: Topics in Fluid Mechanics

As the exam was the first from a new lecturer, it was set slightly straightforwardly. The marks were consequently at a high level, but I did not get the sense that these were inflated in terms of student capability. The most popular questions were 1 and 2, with only 2 out of 17 scripts attempting question 3 on rotating flows. Question 2 was probably too easy, while the awkward part 1(d) defeated most.

#### C5.9: Mechanical Mathematical Biology

**Q1** Overall this was very popular and implemented well, with a suitable spread of marks as the candidates found each part harder than the next.

**Q2** This question was extremely popular, answered by essentially all candidates, but was found to be difficult. In particular, the calculus of variations proved to be more challenging than perhaps expected though once more there was a good spread of marks to differentiate the candidates.

**Q3** This was a surprisingly unpopular question. Essentially all attempts cleared the preliminary parts of the exam, which was a detailed piece of bookwork requiring careful attention to the lecture notes in this part of the course. From the candidates that had navigated this part of the question, many noticed an analogous strategy could be employed for the final parts of the question and typically produced very good attempts.

#### **C5.11:** Mathematical Geoscience

Q1: This question was quite popular and attempted by the majority of candidates. Part (b) was answered too descriptively by some, who failed to give approximate solutions to the model. Quite a few people wrote down the nonlinear ODE satisfied by the temperature for  $t = O(\delta)$  but did not notice that the initial condition had it in the stable steady state of this equation so that the solution was simply constant. Parts of (c) were done well, but no candidates appeared to gain a full understanding of what the phase plane looked like.

Q2: This question was the most popular but also found to be the most challenging. Most candidates picked up marks in part (a), although the non-dimensionalisation was very confused in many cases. The first part of (b) was straight from an example given in lectures, but proved more challenging than anticipated; many candidates failed to realise that the only characteristics with useful information on all came from the origin. The final bed profile was found by only a couple of candidates, a common difficulty being to realise that erosion only starts at each given x when the flood front passes. Part (c) was challenging and was not completed by anyone, though there were some valiant attempts.

Q3: This question was attempted by many fewer candidates but actually turned out to be the highest scoring. Most people who attempted the question got most of part (a) and (b), though many put the glacier surface at z = h in part (b) which led to difficulties deriving the correct expression for  $\tau_b$ . The derivation of the condition for stability in (c) proved surprisingly challenging. A common misconception in the last part of (d) was to say that the steady state loses stability at  $\lambda_c$ , rather than that it simply disappears entirely (so there is no ice sheet).

# C5.12: Mathematical Physiology

The most popular question was 2, followed by 3 and then 1. The straightforward parts were generally well done, but the slightly challenging parts were, well, challenging.

#### C6.1: Numerical Linear Algebra

Most candidates attempted question 1 on matrix factorizations, with a range of scores. Few saw the point of the final part (f) where in particular calculation of the smallest singular value was only correctly done by very few. Too many candidates were happy to query that  $L_1U_1 = L_2U_2$  implies  $L_1 = L_2, U_1 = U_2$  in part (c) without adequate proof.

Question 2 on stationary (simple) iteration was attempted by just over half of the candidates with a range of scores including one full marks. In the final part (c) too many candidates were too quick to introduce  $B^{-1}$  thereby making in almost impossible to apply simple diagonal dominance arguments.

Question 3 on Krylov subspace methods was attempted by under half of the candidates, but attracted (in general) higher marks. Again the final part (c), though well done by some, caused difficulty.

### C6.2: Continuous Optimisation

No report.

#### C6.3 Approximation of Functions

The average raw marks on problems 1, 2, and 3 were 18.9, 20.4, and 15.8. Possibly the exam was easier than average but I don't think this is the whole explanation of these relatively high marks for I was also struck with the good quality of all but one of the papers.

*Problem 1.* Almost everybody did this problem. Most parts had good results except the second half of part (d), worth 5 marks, concerning continuity of the best approximation operator. Only one or two students managed this.

*Problem 2.* About half the students did this problem. They did reasonably well, with part (e), concerning the notion of "Laurent order of accuracy," proving the most challenging.

Problem 3. About half the students did this problem. I was surprised how well they did on parts (a)–(d) — they must have studied these formulae well! I believe nobody got part (e), showing that a certain rational interpolant has no poles in [-1, 1]. It's an easy argument but we hadn't done it in the lectures, reading, or classes.

#### C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question revealed a good spread of abilities across the students who attempted it. In Q1 (a) (iii), surprisingly few students succeeded in sketching the basis functions for various finite elements in one dimension. In Q1 (b) (i) almost all students who attempted the question successfully calculated the basis functions, but few realised for (ii) that the values of the shared degrees of freedom at vertices were not sufficient to constrain the function value, and hence the finite element is not  $C^0(\Omega)$ -conforming. In Q1 (c), many students succeeded at the unseen task of proving unisolvence in (i) and establishing relations between the finite element subspaces in (ii), but most struggled to identify a suitable function in (iii).

Q2: This question was very popular, with every candidate attempting it. Most did very well in Q2 (a), as the material is familiar from past examinations. Q2 (b) was unseen, and succeeded in distinguishing the strongest students. Surprisingly few students realised that uis vector-valued, and hence one must take a dot product of the equation with a vector-valued test function v in (b) (i). Even fewer could successfully apply integration by parts to move the gradient onto the test function. Some students confused the operator -graddiv with -divgrad, and proposed the familiar bilinear form for the Laplacian. Remarkably, every candidate claimed to prove that their bilinear form was an inner product, even when the (incorrect) bilinear form was not in fact an inner product. Strangely, in Q2 (b) (ii) several students ignored the hint, while others did not use the properties of the finite element in question, neglecting to note that the surface integral terms over each cell must cancel due to continuity of the normal component of the function.

Q3: This question was for the most part well-answered. Students who had grasped the central convergence theorems of the course did well in Q3 (a), which was identical to a question on a problem sheet. Q3 (b) was in general well-answered, with minor slips in the definition of  $H_0^2(\Omega)$  or giving a suitable finite element (several candidates proposed the Hermite element, which is not  $C^1(\Omega)$ -conforming in two dimensions). The relative familiarity of (a) and (b) were compensated by the difficulty of (c), which required the use of the Sobolev embedding theorem (or at least knowledge of the integrability of  $H^1(\Omega)$  functions in three dimensions).

# C7.4: Introduction to Quantum Information

#### Question 1

Well done question. Some students struggled with part (b) and the calculations in part (d). Many students failed to provide physical interpretation of the results in part (g).

#### Question 2

This was the most popular question on the paper. Parts (a) and (b) were standard problems and did not pose much difficulty. Some students failed to spot the linearity of equations in part (c). In part (d) most marks were lost for not estimating the imaginary part of the trace. Good attempts at part (e).

#### Question 3

The bookwork in parts (a) and (b) caused no problems. Most marks were lost in parts (c) and (e).

#### C7.5: General Relativity I

Q1: This was seen to be the hardest question on the paper and was the least popular. Parts a to c were mostly bookwork and well done. Candidates had difficulty proving the Bianchi identity for the field strength in part d. Most candidates were unable to complete part f, and there were no correct answers for part g.

Q2: Parts a and b were mostly well done, except for a few algebraic errors. Some candidates used the Euler-Lagrange equations for part a but did not state why this was equivalent to varying the action. In part d, a handful of candidates were able to use the conserved quantities to identify the new coordinates, though few correctly identified the region of (T, X) plane covered by the old coordinates.

Q3: This was the easiest question on the paper and attracted the most attempts. Candidates lost marks in part a by not specifying spherical symmetry and in part b by not showing that the Lagrangian is constant on the geodesic. Part d was attempted by many but completed correctly by few. Parts e, f and g were well done by those who attempted them.

#### C7.6: General Relativity II

Question 1: The question was attempted by less than half of the candidates. Part a) was done very well in general. Most of the candidates wrote down the correct definitions. In part b), many candidates explained well why the metric given in the problem is the most general form of the spherically symmetric metric. Several candidates attempted the coordinate transformations but most of them could not reach the conclusion except for few candidates. In part c), most of the candidates only showed B = B(r). None of the candidates proved that the equation also satisfies  $B_{rr} = 0$ . Question 2: It is pleasing to see most of the candidates can compute Christoffel symbol of a spherically symmetric metric. It is also observed that most of the students had difficulty with straightforward but long calculations, for example, computing the components of Ricci tensor of Vaidya metric. Most of the students had managed to compute just only one non-vanishing component of Ricci tenor, although the question was meant to calculate all components. Also, all examinees taking question 2b) had not shown the vector field tangent to the out-going energy flux in Vaidya space-time obeys null geodesic equation. Some students had not understood the definition of a space-like hyper-surface properly, as attempts were made to show that the normal vector of such a hyper-surface is space-like.

Question 3: Nearly all students attempted this question. Most students did well on the bookwork parts a) and b). Many candidates came up with a correct strategy to solve part c), but nearly everyone struggled with the longer computation that requires correct manipulation of the metric components of Kerr. The first part of d), finding  $\lambda_+$  such that  $\eta_+$  is null on the horizon  $r = r_+$ , was worked out correctly by a good proportion of the candidates, however nearly no one showed that  $\eta_+$  is time-like for all  $r > r_+$ . Finally, part e) seemed easier again for most of those candidates who attempted it and most of those scored good points here.

#### **C8.1:** Stochastic Differential Equations

Question 1 was the most popular, every candidate attempted it. Most candidates realised that Optimal Stopping can be used in 1b, and most managed to solve 1(c)(i); however, few made progress on 1(c)(ii). Questions 2 and 3 were approximately equally popular. While nearly everybody made progress on Question 2(b)(i) many forgot to apply BDG, and 2(b)(ii) turned out to give many candidates trouble but was needed to solve 2(c). For Question 3, the most common mistake was to try to apply Girsanov (this is not in itself wrong but does not help in solving the question); few realised that refining the filtration with  $B^d$  simplifies the conditional expectations.

#### **C8.2:** Stochastic Analysis and PDEs

Most candidates produced good answers to most parts but there were still a couple that struggled to get beyond the basic bookwork.

Question 1: This question was attempted by all candidates. The standard bookwork was generally well done as well as the properties of the resolvent. For the final part, many knew what the domain of reflected Brownian motion should be but were unable to prove it.

Question 2: This was also a very popular question and saw a wide range of marks. No candidate was able to correctly state the conditions for the convergence of a sequence of continuous time Markov chains to a diffusion. The convergence part was mainly well done, though a few candidates used the discrete time Theorem and ended up with the wrong diffusion term. There were a number of good attempts at the final part, with most able to see what the conditions for equilibrium were.

Question 3: The final question was attempted by few people and none managed to get all the parts out. The first two parts were bookwork. The last part proved challenging.

#### **C8.3:** Combinatorics

Question 1 This was the most popular question on the paper. Candidates generally answered the bookwork quite well in (a) and (b), although many forgot the easier direction of (b). There were two different solutions given to (c), one via Dilworth's theorem and the second more directly combinatorial. Few students verified that the poset axioms were satisfied when using Dilworths theorem in (c) or (d)(i). Lastly, only a small number of students found a family  $\mathcal{B} \subset \mathcal{J}$  as in (d)(ii) – the smallest example occurs with  $|\mathcal{B}| = 5$ where  $ints(\mathcal{B}) = disj(\mathcal{B}) = 2$ .

Question 2 Part (a) was answered essentially perfectly by most candidates, being bookwork. Relatively few solved (b) though, which was surprising given it was quite a direct application of the Sauer–Shelah theorem. Question (c) was well answered, being a mixture of bookwork and problem sheet material. Some candidates ignored the instruction to *deduce* LYM from the two families and gave different proofs. A small number of students obtained the upper bound in (d)(i) using that if  $\mathcal{A}$  shatters S then  $2^{|S|} \leq |\mathcal{A}|$  and Sperner's theorem. A handful gave a construction for (d)(ii), although some gave useful ideas.

**Question 3** Both (a) and (b) were mostly answered correctly, with some small details missing in (a)(i) (why restricting pairs  $\{A, A^c\}$  is enough) and (b) (why we can assume a *t*-intersection exists in  $\mathcal{A}$ ). Candidates seemed to find (c) quite difficult – some got partial marks for the case s = 2, but no one completely solved this part (a 'degrees of freedom' argument similar to (b) was possible). About half of the students spotted the trick that allowed the Modular FW in (d) to be used to solve (e) – a similar idea was used in a lemma in the proof of the Borsuk counterexample in the notes.

#### **C8.4** Probabilistic Combinatorics

Question 1 was on the difficult side, although it functioned well in terms of generating a spread of marks. Part (a) was mostly well done. You can write the bookwork proof but with significantly simpler formulae in the special case - or (as most did) write the bookwork proof for general p and then take p = 1/2. The second half of (b) caused trouble for most. (c) is very close to a standard problem sheet question and was mostly (but not always) OK. (d) was not well done.

Question 2 was on the easy side, with most marks ending up similar. In (b) the hard part is to explain the  $\operatorname{aut}(H)$  factor. In (c) some candidates failed to use parts (a) and (b), starting from scratch. Many candidates missed that a 1-edge overlap does not cause dependence in this context. The point of (d) is that pairwise independence is not the same as independence as a set of events. But showing that the events are not independent as a set does not imply that the distribution isn't binomial; to get full marks this needs to be shown by, e.g., noting that we can't have all but one triangle monochromatic. One candidate correctly pointed out that the formula in (b) is incorrect if H has 0 edges, gaining a bonus mark.

Question 3 was only attempted by a few candidates and was in general not very well done. Part (b) in particular turned out to be difficult. The key idea is to observe (with a calculation to back it up!) that two large components of G are very unlikely not to be joined by an edge of H. Overall, this topic (the phase transition) is around 1/4 of the course, so one should expect a question on it in most years!

# **Statistics Units**

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC6 - Graphical Models SC7 - Bayes Methods SC9 - Interacting Particle Systems SC10- Algorithmic Foundations of Learning

# **Computer Science**

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science Categories, Proofs and Processes Computer Animation

# F. Names of members of the Board of Examiners

### • Examiners:

Prof. M Kim (Chair)Prof. G ChenProf. H OberhauserProf. P DellarProf. M LackenbyProf. R Jozsa (External)Dr. J Woolf (External)

# • Assessors

Prof. Samson Abramsky Dr Vinayak Abrol Prof. Konstantin Ardakov Dr Anthony Ashmore Prof. Ruth Baker Prof. Charles Batty Dr Philip Beeley Prof. Dmitry Belyaev Prof. Julien Berestycki Dr Lukas Brantner Prof. Helen Byrne Prof. Coralia Cartis Prof. Jon Chapman Dr Sam Chow Prof. Dan Ciubotaru Prof. Samuel Cohen Prof. Vassilios Dallas Prof. Andrew Dancer Prof. Xenia de la Ossa

Dr Jamshid Derakhshan Prof. Chris Douglas Prof. Cornelia Drutu Prof. Artur Ekert Prof. Alison Etheridge Prof. Patrick Farrell Prof. Victor Flynn Prof. Andrew Fowler Prof. Eamonn Gaffney Prof. Martin Gallauer Dr Kathryn Gillow Prof. Alain Goriely Prof. Ben Green Prof. Peter Grindrod Prof. Ben Hambly Prof. Raphael Hauser Dr Matthew Hennessy Dr Andre Henriques Dr Samuel Heroy Prof. Ian Hewitt Dr Chris Hollings Prof. Samuel Howison Prof. Peter Howell Prof. Ehud Hrushovski Dr Daniel Isaacson Prof. Dominic Joyce Prof. Peter Keevash Prof. Frances Kirwan Prof. Kobi Kremnitzer Dr Heeyeon Kim Prof. Minhyong Kim Dr Florian Klimm Dr Robin Knight Prof. Marc Lackenby Prof. Renaud Lambiotte Prof. Alan Lauder Dr Eoin Long Dr Michael Lubasch Prof. Terry Lyons Prof. Philip Maini Prof. Lionel Mason Prof. James Maynard Prof. Kevin McGerty Dr Andrew Mellor Prof. Derek Moulton Prof. Andreas Muench Prof. Vidit Nanda Dr Yuji Nakatsukasa

Prof. Luc Nguyen Prof. Nikolay Nikolov Prof. Harald Oberhasuer Prof. Jan Obloj Dr Neave O'Cleary Dr David O'Sullivan Prof. James Oliver Dr Yi Pang Prof. Panos Papazoglou Prof. Ebrahim Patel Prof. Jonathan Pila Prof. Hilary Priestley Prof. Zhongmin Qian Prof. Oliver Riordan Prof. Alex Ritter Prof. Damian Rossler Dr Ricardo Ruiz-Baier Prof. Melanie Rupflin Prof. Tom Sanders Prof. Alex Scott Dr David Seifert Prof. David Seregin Dr Jan Sbierski Prof. James Sparks Dr Daniel Straulino Dr Rolf Suabedissen Prof. Balazs Szendroi Prof. Jared Tanner Prof. Nick Trefethen Dr Carolina Urzua Torres Dr Richard Wade Prof. Andy Wathen